



# DEVELOPING MATHEMATICAL PROBLEM SOLVING SKILLS WITH INTERACTIVE PROBLEM BASED LEARNING AND DISCOVERY LEARNING

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**Abstract:** The study is a quasi-experimental research which was carried out to develop mathematical problem solving skills with interactive problem based learning and discovery learning based on exe-learning and to find out the effect of the learning model on students' mathematical problem solving abilities. The data analysis technique used was ANCOVA. Linear regression equation for the experimental group I and II were  $Y^{\circ} = 0.165X + 78,545$  and  $Y^{\circ} = 0.141X + 76.062$  respectively. The ANCOVA test results showed that  $F_{count} = 7.255$  is larger than  $F_{table} = 3.15$ . As a result, H  $_{o}$  was rejected and H  $_{a}$  accepted and it meant that the two learning models have a significant effect on students' mathematical problem solving skills. The effect of the interactive problem based learning model and discovery learning models toward problem solving skills were 42.5% and 37.7% respectively. It is concluded that the two interactive learning models are effective in developing student problem solving skills.

**Keywords:** Discovery learning, Exe-learning, Interactive PBL, Mathematical Problem solving Skills.

Abstrak: Penelitian ini bertujuan untuk mengetahui apakah terdapat perbedaan kemampuan pemecahan masalah matematis antara siswa yang diberi problem based learning dengan siswa yang diberi discovery learning dan untuk mengetahui besar pengaruh model pembelajaran terhadap kemampuan pemecahan masalah matematis siswa. Jenis penelitian ini adalah eksperimen semu. Teknik analisis data yang digunakan adalah uji normalitas, uji homogenitas dan uji hipotesis ANOVA. Diperoleh persamaan regresi kelas eksperimen I adalah  $\acute{Y} = 0.165X + 78,545$  dan persamaan regresi kelas eksperimen II adalah  $\acute{Y}=0,141X+76,062$ . Hasil uji ANCOVA menunjukkan F<sub>hitung</sub> yaitu 7,255 dan F<sub>tabel</sub> adalah 3,15. Karena Fhitung F<sub>tabel</sub> sehingga H<sub>0</sub>ditolak dan H<sub>a</sub> diterima sehingga diperoleh kesimpulan bahwa kedua model pembelajaran berpengaruh signifikan terhadap kemampuan pemecahan masalah matematis siswa. Besar pengaruh model problem based learning adalah 42,5% dan model discovery learning adalah 37,7% terhadap kemampuan pemecahan masalah. Ini dapat ditarik kesimpulan bahwa terdapat perbedaan kemampuan pemecahan masalah matematis antara siswa yang diberi problem based learning dengan siswa yang diberi discovery learning.

**Kata kunci:**, Kemampuan Pemecahan Masalah Matematis, *Problem Based Learning, Discovery Learning*.

### INTRODUCTION

As a result of the Industrial development (ID) 4.0, Indonesia has introduced the 2013 National Curriculum which is intended to meet the millelial generations (Nainggolan et al. 2019) who have digital competency and highly intelligence (Shahroom & Hussin, 2018). Recently, computers and internets have been widely used in

education and integrated with learning management system (LMS) software based on the new educational paradigms such as the GeoGebra, eXe Learning, Modellus, Geometric Cabri and Maple which contribute to the teaching and learning achievements (Mussoi and Flores et al. 2011). They offer a number of advantages due to the development of virtual reality in constructing

simulations and animations that helping students to comprehend the basic principles of Basic Sciences. The two GeoGebra and eXe Learning (eLearning XHTML editor) software are free, so they can be used by teachers to create educational planning online and offline. The two software have successfully used in constructing learning activities in physics and Mathematics (Mussoi and Flores et al. 2011).

response to the development of technology Indonesia has developed e-dukasi.net which is an Open Educational Resources (OER), to facilitate sharing of online learning materials used by primary, junior and secondary schools (Barbour, M., Brown, et al. Waters, L.H. 2006). Furthermore, Indonesia, The United Kingdom, British Columbia, Italy, Finland and Slovenia have integrated technology and e-learning into educational system (Barbour, Brown, R., Waters, L.H. et al. 2006)... Due to the rapid development of ID 4.0 students face a lot of problems in daily life therefore it needs to find out a new way to solve the problems developing student problem solving skills.

The problems usually emerge when a person cannot accomplish a specific target or hardly unlikely to accomplish the objectives. The problem is generally referred to a relation between the subject targeted circumstances. and Furthermore, problems are defined as obstacles or barriers toward objective accomplishments (Yavuz, Arslan & Gulten, 2010). A person who can solve problems in daiy life he/she succeeds in different fields (Ekici & Balim. 2013). As a result, it should be overcome by any means in order to achieve the goals, since it can be seen as difficulties faced by people in their life.

When a person identifies the problems then he/she should determine objectives in order to solve the difficulties so that he/she may get away from the obligation. The problems can

be solved by the students by motivating them. Teachers should motivate the students to certain extent so that they will be exicited and energized to solve the problems. The teacher should lead them to solve the problems so they would understand the important of the problem and comphrehend difficulties that prevent them from accomplishing the objectives. problem contains a conflict or difficulty that should be solved during teaching and learning processes. However, the difficulty should be clearly known by the students so that they are motivated to solve the problems for their satisfaction. Problem-based learning (PBL) generally carried out in two stages i.e through collaborative and self-directed learning processes (Yew et al. 2011). It was found that PBL enhanced student achievements (Yew et al. 2011). Therefore, solving the problems is considered as the main way of sustaining daily life, people work hard to solve the problems as soon as possible (Tambychik & Meerah, 2010). Usually, people who face the problems would solve the problems through problem solving skills (Güven, 2010). It was found that mathematics was successfully solved with problem solving skills (Ciffarelli et al. 2010).

Problem-based learning has long-term effects student on achievement in mathematics conventional strategy (Crowly, 2016). In problem-based learning (PBL), students usually work in group collaboratively to determine what information and data needed to solve the problems. They solve the problems using the new and they knowledge retain knowledge for long-term and it can be used to solve other problems in different areas. In this case, teachers help the students to facilitate the learning process and not to give knowledge directly. Therefore, PBL is intended to help develop knowledge, students to problem-solving skills, individual learning skills, collaborative skills and

intrinsic motivation (Hlmelo-silver et al. 2012).

Polya (2015) introduced solving problems stages ie. the first step is to understand the problem comprehensively. Therefore. the teachers should ask students whether they understand the problems. The second step is to devise a plan in order to find reasonable ways to solve the problems by considering the relationship between data and the previous knowledge. The third step is to carry out the plan patiently and persistenly according to the plan. Finally, the fourth step is to look back at what you have achieved and what have'nt. Yew (2011) found that PBL is influenced by student previous knowledge.

## **OBJECTIVES**

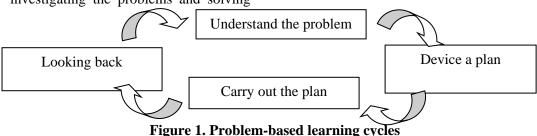
The investigation is intended to develop mathematical problem solving skills using interactive problem-based learning and discovery learning models based on exe-learning.

# LITERATUR REVIEWS Problem solving skills

According Collins to Dictionaries (2014) a problem is defined as unsatisfactory circumstances causes people unsatisfied and it needs to resolve. Solving problem needs skills so that the person satisfied. Problem solving skills help students in solving the problems they meet in daily life (Armağan, Sagir & Çelik, 2009). As it is known that problem solving is defined as solving the problems using problem solving skills involving cogmitive and affective skills (Serin, Serin & Saygih, 2009). Usually, the problems can be divided into defining the problems, investigating the problems and solving the problem by finding information and data from internet, library and other resources (Caliskan, Selcuk & Erol, 2010; Ekici & Balim, 2013). In order to solve problems, teachers need to elaborate problem solving skills to students so they would be able to solve the problems (Yavuz, Arslan & Gulten, 2010; Lee, 2010). Cifarelli et al. (2010) reported that students should be taught problem solving skills, in order to help them solving the problems academically.

The problem solving skill is the main objective in mathematics education curricula because it is needed to solve mathematical problems (Pascual & San Pedro, 2018). As a result they would get empathy, compromise and share responsibility in making decision (Armatana et al. 2009).

Α student who mastered problem solving skills is usually highly self-confident, creative and thinking independently (Ozrecberoglu, Caganaga, 2018). Students who have problem-solving abilities are successful in solving problems than student who has manipulative skills. Therefore, problem solving could be seen as a tool to enrich student understanding and knowledge mathematics (Kaur., Har, & Kapur. 2009). During the problem solving process the students are enchanced to reflect on their own ways of thinking and habits, curiocity and confidentce so that they could apply the strategies into other subjects. It was found that problem solving skills affected achievements of female students (Arma÷ana et al. 2009). Furthermore, Polya (2015) introduced cycles in problem-based skills as shown below



In order to carry out the PBL effectively, Polya (2015) introduced four stages in problem-based learning process as shown in the following table 1

Table 1. Problem-based learning phases

phases					
Phases	Teacher behavior				
Phase 1:	Discusses the objectives				
Introduce the	of learning.				
problem to	Explain various				
students	important logistical				
	needs,				
	Motivates students to				
	engage in problem-				
	solving activities				
Phase 2:	Helps students to				
Directing	explain and organize				
students to	the problem learning				
investigate	activities.				
Phase 3:	Motivates students to				
Guiding the	collect the right				
students to	information, carry out				
investigate	experiments and look				
individually	for explanations and				
and groups	solutions				
Phase 4:	Helps students in				
Develop and	planning and preparing				
deliver the	appropriate artifacts,				
findings.	such as reports,				
	videotapes and models				
	and shares the				
	assignment with				
	classmates.				
Phase 5:	Evaluates student				
Analyzing	learning outcomes and				
and	reflect on their				
evaluating the	investigations and the				
PBL process	processes they use				

# **Discovery Learning**

Discovery learning is referred to an inquiry teaching method which is used to find new facts and information based on previous knowledge and experiences. Discovery learning has been introduced into educational system for over 14 years to help students to find out why student's achievement different to some extent. It was known that students have greater comprehension of such knowledge since they discovered knowledge through discovery learning (Mukherjee, 2015). It was found that discovery learning, increased performances student based perceptual and cognitive components (Raab. et all, 2009). In addition, it was found that discovery learning tasks incorporated with computer-based simulations increased student outcomes (Dalgarno, Kennedy& Bennet, 2014). Furthermore, Saab et,al (2009) found that discovery learning increased student motivation and performances positively. In discovery learning student manipulate structure and transfer information in order to find new facts and information. (Prasad, 2011). The learning method encourages students to discover facts, correlation, and new knowledge based esperiences their previous and knowledge. They use intuition. imagination and creativity to find new information from library, web and peers (Pappas. 2014).

It is also known that the discovery learning has advantages such motivating students, actively involving students, promoting student's independency and autonomy, and higher levels of knowledge retention. However, it needs time to prepare a well discovery learning model as a result it should not be used as a main instruction method since it has limitations in practice (Pappas, C. 2014). By discovery learning the students will enrich their knowledge and comprehension of any subject, they will retain the subject longer and will be able to transfer that new knowledge easier to other learning circumstances and without discovery learning the students rely on memory and abstract thought and will be bored and have no fun. Balim (2009) found that discovery learning increased student success in science and technology. It is due to the fact that discovery learning makes students actively participate in learning process. Science teaching usually bases on the students understanding of natural science and phenomena discovered by inquiring

based on experiments. Balin (2009) found that discovery learning method increases student success since it motivates students to actively participate in the learning process. Furthermore, Mfon Effiong (2010) found that discovery learning is effective in transferring scientific concepts to students. Furthermore, Saleh (2018) found that discovery learning improved pre-academic mathematical concepts of disability children.

According to Pappas (2014), the discovery learning consists of five stages i.e. solving problem, managing, integrating and connecting students to the real world and giving feedback (see Table 2.). In solving the problems, teachers should gude and motivate students to find out solutions by integrating previous knowledge with new information. Therefore, the students should actively participate in solving the problems.and the teachers allow them to work in groups or individually. Then, the teachers motivate them how to integrate previous knowledge with new knowledge, and motivate them to relate the new knowledge with the real world. The students should analyze and interprete the new information. At last, the teachers should take responsibility to give feedback to the students (Pappas. 2014).

Table 2. Discovery Learning Model Stages (Pappas, 2014)

Stages (1 appus, 2 s	Teacher Roles				
Stage 1.	Guide and				
Provlem solving	motivate				
	students to find				
	out solutions by				
	combining				
	previous				
	knowledge and				
	recent				
	information to				
	resolve the				
	problem.				
	Students should				
	actively				
	involved in				
	solving the				

~ -	problems.
Stage 2.	Allow students
Learner	to work
Management	individually or
	in groups, and
	work at their
	own pace. As a
	result, the
	students feel
	relieve from
	unnecessary
	stress or
	burdern and
	makes them
	satisfied.
	satisfied.
Stage 3.	Teach students
Integrating and	how to
Connecting	incorporate
	previous
	knowledge
	with new
	information,
	and inspire
	them to hook
	up to the real
	life. Simple and
	familiar
	knowledge
	motivate
	students to
	study further
	and discover
	something new.
Stage 4.	Teachers
Information	encourage
Analysis and	students to
Interpretation.	analyze and
interpretation.	internalize the
	information
	•
	and not to memorize the
	intended keys.
Stage 5.	Teachers give
Failure and	feedback to
Feedback	students, so
	that they know
	how far they
	have mastered
	the subjects

### RESEARCH METHODOLOGY

This is a quasi-experimental research which is intended to find out the effectiveness of interactive problembased learning and discovery learning models in developing mathematical problem solving skills

Two classes of grade VIII student were selected by random sampling technique and divided into 2 grouped namely experimental group I and experimental group II which consisted of 32 students respectively. The group I was taught by interactive PBL and group II by interactive discovery learning models..

Data collected by using teacher made test which was intended to find out level of students mathematical problem solving skills. The test was carried out two times i.e. pre-test  $(T_1)$  and post-test  $(T_2)$ .

The data were analyzed by Ancova and Normality test was used to see whether the sample is normally distributed or not. This test was designed by comparing the cumulative distribution values of empirical data with the expected normal distribution. When the p value is not significant, it

means that there is no difference between the two distributions. Then homogeneity test was used to find out whether or not the population variance was homogen. If the data variance are the same then the sample is said conversely it is homogen, heterogen. If the sample tested normally distributed then the homogeneity test was carried out. In this study, inferential statistical analysis was used to test the hypothesis. Because the data collected in the study consisted of pre-test scores treated as independent variables and students' problem solving skills as dependent variables, the data were analyzed using co-variance analysis (ANCOVA) and the coefficient of determination index.

### RESEARCH DESCRIPTIONS

Before conducting the research, pre-tests were carried out for the two experimental groups in order to find out student initial knowledges then the experimental group I and II were taugth with the interactive PBL and Discovery Learning respectively The statistical description of the pre-test scores shown in the table 3 below.

**Table 3. Pretest Descriptive Statistics** 

	•		•	•	•	•	Std.	
	N	Range	Min	Max	Mean		Deviation	Variance
						Std.		
	Statistic	Statistic	Statistic	Statistic	Statistic	Error	Statistic	Statistic
pretest1	32	50,00	40,00	90,00	64,218	2,0219	11,43773	130,822
pretest2	32	45,00	37,50	82,50	61,562	2,3566	13,33123	177,722
Valid N (listwise)	32							

It was found that the pretest average value of the students were considerably similar i.e. 64.22 and 61.56. The low average scores of the pretest in both classes showed that there was a need to find out the effect of the interactive PBL and discovery learning models based on Polya stages.

The average pretest scores of the experimental group I and II were shown

in table. 3 above, showed that the two groups were considerably similar. After conducting the pretest, the experimental group I was taugth with interactive PBL and the experiental group II taugth with discovery learning models. Then, they were given a post-test at the end of the class. The post-test was carried out in order to find out student problem solving skills.

**Table 4. Postest Descriptive Statistics** 

							Std.	
	N	Range	Min	Max	Mean		Deviation	Variance
						Std.		
	Statistic	Statistic	Statistic	Statistic	Statistic	Error	Statistic	Statistic
Posttest group 1	32	18,00	80,00	98,00	89,062	,91133	5,15525	26,577
Posttest test group 2	32	24,00	70,00	94,00	84,750	1,0952	6,19573	38,387
Valid N (listwise)	32							

Based on the data in the table 4 it showed that the experimental group I had a higher score than group II.

### **DATA ANALYSIS**

Normality test was carried out in order to find out whether the data comes from normal sample a distribution population or not and the showed that the significance value of Asymp. Sig (2tailed) in group I and II were 0.200 and 0.161 larger than 0.05 respectively, so samples were normally that the distributed.

It was found that the posttest significance value of Asymp. Sig (2-tailed) in group I and II were 0.200 and 0.106 respectively larger than 0.05, so that the samples normally distributed. Then, Lavene's homogeneity test at a

significant level of 5% was acrried out in order to find out whether the sample comes from a homogeneous population or not.

Based on the Lavene's test, the significance value "Based on Mean" or based on the average was 0.175 larger than 0.05, it means that there is no difference in variance between the pretest data in the group I and group II. In other words the samples waere homogenously distributed.

Then, a homogeneity test was carried out for the post-test data,a and found that the significant value of "Based on Mean" was 0.649 larger than 0.05, so that it is concluded that there is

no difference in variance between the post-test data in the two classes. In other words the sample was homogeneously distributed.

# HYPOTHESIS TESTING BY ANCOVA

### **Determination of linear regression**

Regression equations were carried out to find out the effect of the two models on student problem solving abilities. The regression equation coefficient was calculated with SPSS for Windows version 22 software at a significant level of 5%. It was found that regression equations were  $(Y 1)^{\hat{}} =$  $0.165X \ 1 + 78,545 \ \text{and} \ (Y \ 2)^{} =$ 0.141X + 76.062 for group I and II respectively. For group I, It was found that  $t_{count}$ =12,271 was larger than  $t_{table}$ = 1,699 so that H<sub>0</sub> was rejected, as a result it was concluded that the regression coefficient was significant.

Then for group II, It was known that  $t_{count}$  =10.182 and  $t_{table}$ =1.699. Since  $t_{count}$ > $t_{table}$ , then  $H_0$  was rejected, therefore, it is concluded that the regression coefficient was significant

### LINEARITY TEST

The linearity test was performed using SPSS version 22 for windows through analysis of variance (ANOVA). The regression linearity test results for group I shown in the following table 5

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Table 5. Liniarity test of the group I ANOVA<sup>a</sup>

		Sum	of			
Model		Squares	df	Mean Square	F	Sig.
1	Regression	40,758	1	40,758	1,561	,221 <sup>b</sup>
	Residual	783,117	30	26,104		
	Total	823,875	31			
a. Depe	endent Variabl	le: postes1				
b. Pred	ictors: (Consta	ant), pretes1				

Based on the linearity test results above, it was found that  $F_{count}$ =1.561 and the value of  $F_{table\ (0.5)\ (1,\ 30)}$  = 4.17. Since  $F_{count}$  < $F_{table}$ , therefore  $H_0$  was accepted. It means that the group I regression equation was linear. Then linearity test results for the group II were shown in the following table 6.2 below.

**Table 6. Linearity Test of the Group II** ANOVA<sup>a</sup>

		Sum	of			
Model		Squares	df	Mean Square	F	Sig.
1	Regression	109,723	1	109,723	3,047	,091 <sup>b</sup>
	Residual	1080,277	30	36,009		
	Total	1190,000	31			

a. Dependent Variable: postes2

Based on the leanity test results above, it was also found that  $F_{count} = 3.047$  and the value  $F_{table\;(0.5)\;(1,\;30)} = 4.17$ . Since  $F_{count} < F_{table}$ , therefore  $H_0$  accepted. It means that the group I regression equation was linear.

## HOMOGENITY TEST OF THE REGRESSION EQUATIONS

The regression homogeneity test was carried out used SPSS for Windows vers. 22. Homogeneity tests of the regression coefficient were carried out as a condition for conducting co-variance analysis tests. The results were tabulated in the following table 7.

Table 7. Levene's Homogenity test Dependent Variable:

F	df1		df2	Sig.	
,488		1	62	,487	

Based on the homogenity test above, it is known that value of  $F_{count}$ =0.488 and the value  $F_{table~(0.5)~(1,~62)}$ = 4,000. Since  $F_{count}$ < $F_{table}$ , therefore  $H_0$  accepted and as a result, the regression equation was considerably homogeneous and met the co-variance analysis requirements.

### **CO-VARIANS ANALYSIS**

Covariance analysis was carried out due to the fact the regression equation met the linearity and homogeneity tests. It was carried out in order to find out the significant effect of the learning models on students' problem solving abilities and the co-variance analysis results were listed in the following table 6.4 below.

b. Predictors: (Constant), pretes2

**Table 8. ANCOVA Test** 

Tests of Between-Subjects Effects

able: postest						
Type III Sum of Mean						Eta
Squares	df	Square	F	Sig.	Squared	
444,141 <sup>a</sup>	2	222,071	7,255	,001	,192	
14587,470	1	14587,470	476,537	,000	,887	
250,953	1	250,953	8,198	,006	,118	
146,579	1	146,579	4,788	,032	,073	
1867,296	61	30,611				
485684,000	64					
2311,438	63					
	Type III Sum of Squares  444,141 <sup>a</sup> 14587,470  250,953  146,579  1867,296  485684,000	Type III Sum of Squares df  444,141 <sup>a</sup> 2  14587,470 1  250,953 1  146,579 1  1867,296 61  485684,000 64	Type III Sum Squares         of Mean Square           444,141a         2 222,071           14587,470         1 14587,470           250,953         1 250,953           146,579         1 146,579           1867,296         61 30,611           485684,000         64	Type III Sum Squares         of of Square         Mean Square         F           444,141a         2 222,071         7,255           14587,470         1 14587,470         476,537           250,953         1 250,953         8,198           146,579         1 146,579         4,788           1867,296         61 30,611           485684,000         64	Type III Sum Squares         of of Square         Mean Square         F Sig.           444,141a         2 222,071         7,255 ,001           14587,470         1 14587,470         476,537 ,000           250,953         1 250,953         8,198 ,006           146,579         1 146,579         4,788 ,032           1867,296         61 30,611           485684,000         64	Type III Sum Squares         of df Square         Mean F Sig. Squared         Partial Squared           444,141a         2 222,071         7,255 ,001 ,192           14587,470         1 14587,470 476,537 ,000 ,887           250,953         1 250,953 8,198 ,006 ,118           146,579         1 146,579 4,788 ,032 ,073           1867,296         61 30,611           485684,000         64

a. R Squared = ,192 (Adjusted R Squared = ,166)

Based on the Ancova analysis results above, it was found that  $F_{count} = 7.255$  and the value  $F_{table\ (0.5)\ (2,\ 62)} = 3.15$ . Since  $F_{count} > F_{table}$ , then  $H_0$  was rejected. This means that there is a significant effect of the learning models on students' mathematical problem solving abilities.

### EFFECT OF LEARNING MODEL ON PROBLEM SOLVING ABILITY

Calculation of coefficient of determination (R<sup>2</sup>) was required in order to find out the effect of the learning models (independent variables) on students' problem solving abilities (dependent variable). The coefficient of determination results of the two groups were listed in the following table 6.5 and 6.6 below.

Table 9. The coefficient of Determination (R<sup>2</sup>) of the Group I

Model Summary<sup>b</sup>

'				Std.	Error	Change St	atistics				
		R	Adjusted	of	the	R Square	F			Sig.	F
Model	R	Square	R Square	Estima	ate	Change	Change	df1	df2	Chang	ge
1	,652°	,425	,395	5,1250	00	,425	5,924	1	30	,027	
a Pred	ictors:	(Constar	nt) pretes1								

a. Predictors: (Constant), pretes1b. Dependent Variable: postes1

Based on the coefficient of Determination (R<sup>2</sup>) results above, the effect of the problem based learning model of the group I on students' problem solving abilities was 0.425 or 42.5%. The results of the determination coefficient of the group II were tabulated in the table 10 below.

Table 10. The Coeffisient Determination (R<sup>2</sup>) of the Group II

Model Summary<sup>b</sup>

				Std.	Error	Change St	atistics				
		R	Adjusted	of	the	R Square	F			Sig.	F
Model	R	Square	R Square	Estima	ate	Change	Change	df1	df2	Chang	ţe
1	,614ª	,377	,347	5,1000	07	,377	4,847	1	30	,035	
a. Predictors: (Constant), pretes2											

b. Dependent Variable: postes2

As shown in the table 6.6 above, the effect of the discovery learning model of the group II on students' problem solving abilities was 0.377 or 37.7%.

### **RESULTS**

In this study two different learning models were used, namely Interactive Problem Based Learning and Discovery Learnings based on exelearning. The Interactive Problem Based Learning was carried out towards the experimental group I and Discovery Learning towards group II. The number of student samples in group I and II were 32 students respectively. Before conducting the experiment the two groups were tested with pretest in order to find out students' initial abilities. It was found that the pretest average value of student were 64.21 and 61.56 in the group I and II respectively. The average problem solving ability of the students was classified as low criteria.

Having determined the students' initial abilities, the two groups were treated with different learning models. The experimental group I was classified into 8 groups of discussion and taught with interactive problem based learning. Each group discussed the problems given. Afterwrds, the representatives delivered the results of the group discussions. The experimental group II was taugth with interactive discovery learning and during the learning process the student was stimulated to ask questions concerning with the problems in order to draw conclusions of the problems given. At the end of the class a posttest was carried out in order to find out students' problem solving abilities of the two groups. It was found that the posttest average scores of the experimental group I and II were 89.07 and 84.75 respectively. The average problem solving ability of the students was classified as moderately high.

Furthermore, effect of the learning model on students' mathematical problem solving abilities was analyzed by ANCOVA analysis. The regression equations found were  $(Y_{\_1})^{\hat{}} = 0.165X_{\_1} + 78,545$  and  $(Y_{\_2})^{\hat{}} = 0.141X_{\_2} + 76,062$  for the group I and group II. A positive coefficient number means that there was a positive effect of

the independent variable (pretest) on the dependent variable. It means that if the learning model score increases one unit, then students' problem solving skills will increase by 0.165 and 0.141unit for the group I and II respectively. The constant numbers in the equations referred to a significant effect of the learning model on problem solving skills. It was found that the two experimental groups have a significant effect on students' problem solving skills where F<sub>count</sub>=7.255 was larger than  $F_{table}=3.15$ , as a result  $H_0$ rejected and Ha accepted. It means that the learning model has a significant effect on students' mathematical problem solving skills. The effect of the problem based learning model was 42.5% and the discovery learning model was 37.7%. The the regression model in the group I and II was slightly difference (2.483).

### **CONCLUSIONS**

Based on the findings, it is concluded that, mathematical problem solving skills could be developed through interactive problem based learning and discovery learning based on exe-learning. It was shown by the moderately high student posttest scores of 89.06 and 84.75 in the experimental group I and II respectively. In addition, the effect of the interactive problem based learning model on student 'mathematical problem solving skills were 42.5% and 37.70% for the experimental group I and II respectively. In the regression model there was a difference of 2.483 between experimental group I and II. It means that the interactive problem based learning had a higher effect on student Mathematical Problem Solving Skills than Discovery Learning.

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