A RESEARCH VIEW OF THE INTEGRATION OF GRAPHING CALCULATORS INTO THE SCHOOL MATHEMATICS CLASSROOM

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Abstract. Teachers play a key role in the integration of the graphing calculator into the teaching and learning of classroom mathematics. Research points to the influence that knowledge and beliefs of teachers play in the quality of classroom use of graphing calculator. It has been argued that there are five broad categories of teacher response when it comes to the integration of technology within their classroom. Teachers tend to view technology as either: a demon; a servant; an idol; a partner; or, a liberator. These categories are used to discuss a selection of research findings and their implications for graphing calculators while providing possible directions for future researchers investigating the integration of graphing calculators into the teaching and learning of classroom mathematics.

Keywords : Integration, calculator, mathematics and graphing

BACKGROUND

The purpose of using any technology in the mathematics classroom, be it the internet or the graphing calculator, is to provide a context within which students can engage with important mathematical activities.

> Computers and calculators have changed the world of mathematics profoundly. They have affected not only what mathematics is important but also how mathematics is done. It is now possible to execute almost all of the mathematical techniques taught from kindergarten through the first 2 years of college on hand-held calculators. This fact alone must have significant effects on the mathematics curriculum... the changes in mathematics brought about by computers and calculators are so profound as to require readjustment in the balance and approach to virtually every topic in school mathematics. (Romberg, 1992, p.772)

Globally there has been pressure to change the objectives and topics of the mathematics curriculum with less focus on computational skills and more upon exploratory and investigative tasks. There are within the existing research literature at least four dichotomies that reflect aspects of what constitutes important mathematics: mathematical activity, mathematical knowledge, mathematical understanding, and mathematical learning. Firstly, Zbiek, Heid, Blume, and Dick (2007) make a distinction between two different types of mathematics activity: technical and conceptual.

The technical dimension of mathematical activity is about taking mathematical actions on mathematical objects or on representations of those objects. Procedures can then be built out of sequences of mathematical

> objects... Conceptual mathematical activity involves understanding, communicating, and using mathematical connections, structures, and relationships (p. 1120).

This distinction resonates closely with another, made between procedural and conceptual knowledge (Hiebert & Lefevre, 1986). While procedural knowledge is focussed on knowing a pattern of steps or a recipe to obtain a correct answer, conceptual knowledge "is knowledge rich in relationships" (Hiebert & Carpenter, 1992, p. 78). While it is highly desirable to promote mathematical conceptual knowledge it is also "important to emphasize that both kinds of knowledge are required for mathematical expertise" (p. 78). This is also true for technical and conceptual mathematical activity.

In a third dichotomous comparison Skemp (1976, 1979, 1986 1989) distinguishes between instrumental mathematics understanding and relational understanding. The first emphasises understanding how to get a correct answer. "The kind of learning which leads to instrumental mathematics consists of the learning of an increasing number of fixed plans, by which pupils can find their way from particular starting points (the data) to required finishing points (the answers to the questions)" (Skemp, 1989, p. 14). Relational mathematical understanding emphasises the interconnectedness of knowledge and schema construction. "In contrast, learning relational mathematics consists of building up a conceptual structure (schema) from which its possessor can (in principle) produce an unlimited number of plans for getting from any starting point within his schema to any finishing point. (I say 'in principle' because of course some of these paths will be much harder to construct than others.)" (Skemp, 1989, p. 16). The fourth and final comparison refers to the work of Ramsden (1984, 1992) who distinguishes between surface and deep learning where deep learning is complex and interconnected and resonates strongly with relational understanding..

The implications of this brief introduction are that depending upon the dichotomy chosen, the aim of many classroom mathematics teachers would be to use technology (in this case the graphing calculator) to build upon: technical mathematical activity to develop conceptual mathematical activity, procedural knowledge to develop conceptual mathematics knowledge, instrumental understanding to develop relational understanding, and surface learning to develop deep learning. However, not all teachers will share the same aim and the next section will discuss the ways in which classroom teachers have coped with the move to integrate technology into their mathematics classroom.

TEACHER INTEGRATION OF TECHNOLOGY

The process of transforming information into knowledge is a demanding and purposeful process that requires both instruction and a community of learners. In this transformation process the role of the teacher is crucial.

> There are mathematical skills that require practice, and subtle mathematical ideas and concepts a student often finds difficult to grasp. Many a student, and indeed many a professional mathematician, will tell of a particular teacher who had a major influence in the development of their understanding and appreciation of the subject (Hosking, 2002, p. 7).

There has been an explosive growth in the availability of technology for mathematics classrooms during the last quarter of a century, and this growth has been accompanied by an enthusiasm for the potential of new technologies in teaching and learning of mathematics (Zbiek et al., 2007). The manner in which educational systems, schools and teachers have responded to this pressure to change and adapt has manifested itself in a variety of ways. Within the school context, the cost, size and portability of the graphing calculator saw it become a tool taken up in many secondary schools. The impact of the graphing calculators has the potential to lessen the need for technical activity and allow more time for conceptual mathematical activity. Mathematics teachers saw that graphing calculators could influence both the teaching and learning of mathematics. For example: the graphing calculator's ability to instantly provide multiple representations of a function, or the dynamic geometry software that offers older students opportunities to investigate spatial phenomena by modelling mechanical linkages to gain insights into geometric relationships and proofs (Vincent & McCrae, 2000). Zbiek et al., (2007) included graphing calculators among the list of cognitive tools because they helped transcend the limitations of the mind.

Teachers have adopted different approaches in response to their experience with technology and beliefs about the teaching and learning of mathematics using technology. The role of the teacher is most important as,

> several investigations of students using technology that have been carried out over the past ten years show that students do not learn from simply interacting with technology. The design of adequate tasks and the role of the teacher play a critical role in the success of integrating technology (Laborde, 2003, p. 23).

A teacher's beliefs about the teaching and learning of mathematics have a strong influence upon his or her integration of ICT (Baturo, Cooper, Kidman, & McRobbie, 2000). Some claim it depends not only upon teachers' existing beliefs but also upon their readiness to change.

> It is relatively easy to train teachers to use various ICT tools, but it is much harder to help mathematics teachers integrate ICT into teaching and learning. Teachers have to change their conceptions of the nature of mathematics, the aims of mathematics education, and how mathematics is learned (Yang, Butler, Cnop, Isoda, Lee, Stacey, Wong, 2003, p.61).

White (2003) conducted an examination of the research literature and proposed five broad categories to describe the common teacher responses to the integration of technology within their classroom. These metaphors were further expanded (White, 2004) and describe how teachers tend to view technology as either: a demon; a servant; an idol; a partner; or, a liberator. These categories are not meant to be used for labelling teachers but as a way of understanding the diversity of approaches in the classroom towards the use of graphing calculators. In the following sections the categories will be further expanded to include a focus on graphing calculators. It is also acknowledged that while categories help us to think about new things in the context of older more familiar things they can be constraining too. They are limited because all categories break down eventually if interrogated too carefully as there are differences as well as similarities between things.

ICT AS DEMON

White (2004) claimed that evidence for this approach is observable in the teachers who actively oppose and subvert any attempt to integrate technology into the curriculum. These teachers are regarded as the resistance fighters who are either afraid or unwilling to learn and so they conduct an active or passive campaign of resistance. When directed by authority, they do the minimum effort required and this often results in inappropriate use. For example: the

> Singapore Ministry of Education recommends that 30% of curriculum time should involve use of ICT. Some mathematics teachers try to satisfy this by using PowerPoint as a presentation tool, which is usually not effective to teach pupils how to solve problems (Yang, Butler, Cnop, Isoda, Lee, Stacey, Wong, 2003, p.61).

Goos, Galbraith, Renshaw, and Geiger (2000) used the metaphor of technology as master to describe this stage.

The teachers in this category make use of every difficulty to resist the inclusion of the graphing calculator into mathematics assessment. As a consequence, teachers either ban the devices from examinations or attempt to write graphing calculator-free questions. A considerable amount of time is focussed on discussion of the issues of calculator access or the types of graphing calculators that will be permitted for student use. So many conditions are placed on which model can be used that "due to the limitations of devices that are allowed in an examination, it is difficult to design appropriate questions to measure understanding" (Yang, et al., 2003, p.60). Kemp, Kissane and Bradley (1996) developed a typology of the possible relationships between the tasks given to students in examinations or tests and teacher intentions regarding graphing calculators.

Table 1.
Expected usage of graphics calculators and examinations (*Kemp, Kissane & Bradley, 1996)*

Graphics calculators are expected to be used
1. Students are explicitly advised or even told to use graphics calculators
2. Alternatives to graphics calculator use are very inefficient
3. Graphics calculators are used as scientific calculators only
Graphics calculators are expected to be used by some students but not by others
4. Use and non–use of graphics calculators are both suitable
Graphics calculators are not expected to be used
5. Exact answers are required
6. Symbolic answers are required
7. Written explanations of reasoning are required
8. Task involves extracting the mathematics from a situation or representing a situation mathematically
9. Graphics calculator use is inefficient
10. Task requires that a representation of a graphics calculator screen will be interpreted

ICT AS SERVANT

In the next category of White's (2004) model, teachers adopt a conservative position where the integration of technology is limited to just the use of ICT by the teacher and students "yet the pedagogy remains much the same as in the past" (Downes, Fluck, Gibbons, Leonard, Matthews, Oliver, Vickers, & Williams, 2001, p. 26). Thus technology is a tool for enhancing students' learning outcomes but within the existing curriculum and using existing learning processes (Russell & Finger, 2003). Salomon (2000) sees this as a consistent tendency of the educational system to preserve itself and its practices by the assimilation of new technologies into existing instructional practices. It fits into the prevailing educational philosophy of cultural transmission, where there is a body of important knowledge that has to be mastered (p.2).

Generally the focus of teachers is upon student mastery with a consequent development of drill and practice programs, and a "most powerful and innovative technology is taken and is domesticated ... Emasculated tools cannot do any harm, but they do not do any good either" (Salomon, 2000, p. 2). Baturo, Cooper, Kidman, and McRobbie (2000) conducted a study of Queensland teachers examining perceptions towards technology and found that transmission models of teaching were very evident and have been very resilient and resistant to change.

While teachers at this stage cling to current practice, there is evidence that the presence of technology gradually influences these teachers to change. A longitudinal case study inquiry by Kendal, Stacey, and Pierce (2002) studied instructional practices of a teacher using computer algebra systems (CAS). In the early teaching using CAS, the teacher continued emphasising by-hand skills and only employed CAS for demonstrations and solving problems that required difficult by-hand symbolic manipulation. Yet over time she increased her use of CAS as a pedagogical tool for students to learn mathematics via activities requiring exploration and investigation. This shift in teaching style was also evident in changes to worksheets. Early worksheets took advantage of CAS symbolic capacities to look at a wider range of functions than otherwise might have been examined. Later worksheets attended to purposes such as introducing new CAS commands, introducing concepts in different representations, and guiding students to generate examples from which rules might be formulated. This teacher case study illustrates the challenges faced by teachers in introducing software into a classroom. It also alludes to the possibility of a developmental process that teachers may go through. As they become more comfortable and knowledgeable about the software, they are then more comfortable with extending their range of teaching strategies (White, 2004).

One of the ways in which the graphing calculator changes the pedagogy is the increased importance of some mathematics content which demands teacher attention. Mitchelmore and Cavanagh (2000) examined student understanding of function graphs when using graphing calculators. The sample consisted of 25 Australian Years 10 and 11 students from five schools who had used graphing calculators for 6-12 months. The results indicated that students had a weak understanding of the effects of scale-changes on a graph, of numerical accuracy, and accepted graphs as they saw them. The researchers concluded that misinterpretation of calculator graphs and the effects of scale on graphs needed to be explicitly addressed if students were to develop appropriate ideas and strategies for working with the technology. They also reported a tendency of students to accept the graphic image uncritically, without attempting to relate it to other symbolic or numerical information

ICT AS IDOL

This approach in White's model (2004) includes teachers who are seduced by the 'dazzle effect' of these 'techno toys' and who fail to consider the teaching and learning implications beyond a surface level. This approach promotes technology as a tool for use across the curriculum where the emphasis is upon the development of technology related skills, knowledge, processes and attitudes (Russell & Finger, 2003, p. 3). It is more focused upon teaching about technology rather than with technology. Salomon (2000) is critical of this 'techno-centic' approach where educators expect that technology will bring about a change and teachers "promote the use of computers in the classroom as if using technology was an end in itself" (Downes, et al., 2001, p. 23). Evidence of this approach is seen in professional development programs that give teachers intensive use of a specific technology but fail to assist teachers with the teaching and learning implications. These teachers then struggle to integrate what they have learnt when they return to their classroom. Zbiek et al., (2007, p. 1179) use the terms 'instrumental genesis' to describe how technology does not have the same automatic power for all users and how intelligent use requires both conceptual and technical knowledge.

In the beginning when the use of graphing calculators in the mathematics classroom was an option, many teachers were faced with the issue of whether the time spent on student tool familiarity was worth the gains in student understanding and learning. Those teachers caught up in the excitement of using a new tool were sometimes distracted away from the mathematics and early assessment items sought correct button sequences as a measure of student technical mathematics knowledge. However, these should not be discounted as the entering of correct expressions requires a conceptual appreciation for the structure of the expressions being entered. The use of CAS requires technical knowledge of the notations and conventions of CAS (Zbiek et al., 2007)

ICT AS PARTNER

White (2004) in his next category, points to classrooms where "students are actively engaged in gathering data, aggregating their data with those gathered by other students, and making meaning of their results" (Downes et. al., 2001, p. 26). Here teachers have seriously attempted integrating technology into their classroom. Teachers have sought to "change the orientation from teaching *about* computers to teaching *with* computers" (Russell & Finger, 2003). Here technology is integral to the pedagogy which seeks to change not only *how* students learn but *what* they learn. The use of graphing calculators and the various probes allow data to be captured outside the classroom and to be transferred and analysed at a later time. Real data thus can be analysed by the use of a mathematical model, and conclusions and inferences produced.

Hong, Thomas and Kwon (2000) used a pre-post-test methodology with a module of work implemented over four lessons, addressing the solution of linear equations using CAS. It was implemented in a New Zealand Form 4 secondary class of 13-14 year old students. In the pre-test, students scored well on standard questions which could be solved symbolically and there was no significant difference in favour of CAS in performance on those in the post-test. There was a significant difference on questions that were more conceptually demanding and involved relating data between two or more representations. They concluded that one value of graphing and CAS calculators is that they can be used to make explicit links between different representations.

Research indicated that regular access to the technology can have a positive influence on linking different representations of functions. The principal findings of Chinnappan and Thomas (2000) were that teachers who believed in multi-representational approaches used such approaches when teaching and that effective technology use was buttressed by well-developed mathematical knowledge. The early belief that good teaching of functions and calculus was preceded by simultaneously developing the three representations (numeric, graphic and symbolic) has lost favour. Research cautions against the simultaneous adoption of multiple representations as they may need to be introduced over time rather than from the very start, with graphical and symbolic representation taking precedence over numeric representations (Kendal & Stacey, 2003).

The limitations of the graphing calculator can be used as an opportunity for learning, teaching and assessing mathematics. Dick (2007) uses the term 'mathematical fidelity' to highlight that a tool such as a graphing calculator doesn't always represent mathematics as it is understood by the mathematics community (eg., the graph of the sine function with increasingly small periods). He noted three areas in which a lack of mathematical fidelity can occur: discrepancies between tool and mathematical syntax conventions; under-specification in mathematical structure; and, limitations in representing continuous phenomena and discrete structures and infinite precision numerical computations. Hart (1991, as cited in Zbiek et al., 2007, pp. 1175-6) conducted extensive task-based interviews with students using graphing technology with a multiple representation based calculus curriculum. Several tasks contained a lack of mathematical fidelity and Hart examined how the students resolved these apparent conflicts. Hart found the students algebraic skills strongly influenced whether they regarded the machine generated graph as having authority. Hart also found that students with strong algebraic skills using a traditional curriculum without technology were reluctant to use graphical representations even though these provided much more accessible information than a symbolic formula.

Graphing calculators can include dynamic geometry which "helps students in defining and identifying geometrical properties and the dependencies between them, but not in proving them" (Healy, 2000, p. 114). Vincent (2003) conducted a study of a Year 8 secondary school classroom where students worked in pairs on an exploratory task using *Cabri Geometry* to generate and then prove conjectures about quadrilateral properties. She reported that the dynamic environment supported students' argumentation and helped to connect conjecturing with proving. The dynamic feedback from the software (e.g., "dragging" figures to look for invariant properties) and teacher support helped students to construct a written proof. For Healy (2000) the challenge is the "search for learning contexts which help students switch naturally between deductive and inductive concerns - contexts in which it makes sense to formulate statements and definitions through agreed procedures of deduction without severing any connection from empirical justification" (p. 103).

ICT AS LIBERATOR

In the final category, White (2004) proposes a radical approach where integration is a component of the reforms that alter the organisation and structure of schooling itself. "Among the diversity of school types will be virtual schools, where students spend part or all of their time working ‘off-campus’, for example, from home using an online computer" (Russell & Finger, 2003, p. 3). While there are over one hundred virtual schools already existing in the U.S.A., this phase is still the stuff of experimentation, discussion and research. And there are critics of this phase such as Salomon (2000) who states that "not many students have the self-discipline or the sustained motivation to be distance-, virtual learners" (p. 4). Salomon regards this approach as another example of techno-centrism that is in danger of yielding virtual results. He is supported by other researchers who have reported a number of unintended and unwelcome effects of virtual schools such as the example of high range internet users experiencing increased loneliness, depression, anxiety and poorer social relationships (Kraut, Patterson, Lundmark, Kiesler, Mukopadhyay, & Scherlis, 1998).

Yet there are other alternatives to virtual schools where "more sophisticated understandings of the implications of ICTs for reforms in curriculum, pedagogy and assessment are required" (Russell & Finger, 2003, p. 9). Researchers English and Cudmore (2000) investigated the learning of mathematics in communities of inquiry where secondary school students in different countries were networked to participate in shared data-handling investigations. A password-protected extranet on the World Wide Web automatically accepted, processed, and distributed data using Web-based forms. Students generated data by designing surveys that were posted to the extranet for completion by peers from all participating countries. They analysed the data and engaged in statistical problem posing, especially on issues arising from cross-cultural differences. The extranet supported communication and knowledge building in a community beyond the classroom.

Borwein (2005, cited in Zbiek et al., 2007, p. 1170) proposed the following challenges for computer use as (a) gaining insight and intuition, (b) discovering new patterns and relationships, (c) graphing to expose mathematical principles, (d) testing and especially falsifying conjectures, (e) exploring a possible result to see whether it merits formal proof, (f) suggesting approaches for formal proof, (g) replacing lengthy hand derivations with tool computations, and (h) confirming analytically derived results. These challenges could also act as a guide to the use of graphing calculators.

The graphing calculator allows students to quickly generate graphs from algebraic and numerical data. Assessment questions can shift the emphasis to the teaching of higher-order thinking skills such as interpretation rather than the procedural skills of drawing graphs. "Previously, by the time students had drawn a graph, the class was nearly over - now they can talk about what it means" (Yang, et al., 2003, p. 60).

The graphing calculator has the ability to support student engagement in authentic mathematical activity by using the classroom as a knowledge-building community. Collins and Bielaczyc (1997) discusses shared-passions in which students are involved in a collective effort of understanding with an emphasis on diversity of expertise, shared objectives, learning how to learn and sharing what is learned. Learning communities are groups of people who investigate issues and share what they learn with others in the community, thus advancing both their individual knowledge and the community’s knowledge.

The motivational aspects of student use of graphing calculators and the social context have been the focus of Boaler (1993) who examined the factors which influenced a student's choice or reorganisation of mathematical procedures. Surprisingly, these factors were found to be social rather than mathematical in origin and were influenced by the goals that students establish, the contexts in which they interpret them, and has very little to do with the understanding of mathematical concepts. If student behaviour is embedded within social interaction, then what are the implications for graphing calculators? Graphing calculators were designed as personal tools, yet research has reported that students tended to use them as a shared device. Graphing calculators played an important role in group activities as a kind of conversation piece for sharing mathematical ideas and making thought processes publicly available in the classroom. The graphing calculators facilitated social interaction in the classroom because it acted as a common point of reference for students as they discussed their ideas and results (Cavanagh, 2005). Thus the effectiveness of graphics calculators may reside more in this social role than in their ability to reveal mathematical concepts.

CONCLUSION

This paper has sought, through the selective use of research, to inform the current debate on the integration by teachers of the graphing calculator into the mathematics classroom. It has tried to situate the debate within the wider one which focuses upon a greater range of strategies and outcomes than can be delivered through a different view of mathematics teaching and learning.

A view which uses conceptual mathematical activities as well as the technical ones, that develops both instrumental and relational understanding, that produces both procedural and conceptual knowledge, and one that encourages students

to go beyond the surface by deepening their learning. A view which uses conceptual mathematical activities as well as the technical ones, that develops both instrumental and relational understanding, that produces both procedural and conceptual knowledge, and one that encourages students to go beyond the surface by deepening their learning.

The mathematics classroom with the integration of technology such as the graphing calculator is an environment that creates new challenges where creativity is required by teachers, students and researchers. Zbiek et al., (2007) claim that research efforts have lagged behind the acceptance of technology and "as a consequence, research on technology-intensive mathematics and learning has only recently begun to mature into a well-articulated area of scholarship" (p. 1170). Thus it is hoped that the research studies outlined in this paper have the potential to provide an inspiration and a challenge to other researchers to transform their research with the aim of transforming the teaching and learning of classroom mathematics through the integration of technology.

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