TEACHING AND LEARNING MULTIPLICATION OF MULTI-DIGIT NUMBERS IN REALISTIC MATHEMATICS EDUCATION (RME)

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Abstract. This study developed a learning trajectory of the multiplication of multi-digit numbers in Indonesian primary schools using Realistic Mathematics Education (RME). It was the third cyclic process of the developmental research process conducted in 4 primary schools in Medan and Yogyakarta in the academic year of 2000. This study produced a local instruction theory of teaching multiplication of multi-digit numbers in primary schools in Indonesia. The learning trajectory was begun by introducing the repeated addition of ten numbers, the multiplication by 10, the multiplication by multiples of ten, and the standard multiplication algorithm. This local basic learning trajectory theory of multiplication was opened for adjustment and justification and it was functioned as guidance for other researcher in conducting the next developmental research.

Keywords : Mechanistic or algorithm approach, Realistic Mathematics Education (RME), learning trajectory.

BACKGROUND

In primary schools, the mathematics curriculum aims at preparing the students to use and apply their mathematics knowledge and mathematical way of thinking in solving problems in their life and in learning other different knowledge (Depdikbud, 1994). It means that the students should develop their counting ability, enhance their mathematics content knowledge, and structure their attitude to be critique, honest, disciplined, efficient, and effective.

To achieve the goals illustrated above, the learning process of mathematics was conducted by practicing the chalk and talk model or the concepts-operations-example-drilling approach (Suyono, 1996). The teachers explain the mathematics operation and procedures, give some examples, and ask the students to do the other similar problems. This model of teaching is called the mechanistic way of teaching (Treffers, 1987). The teachers teach mathematics with practicing mathematics symbols and emphasizing on giving information and application of mathematics algorithms.

In learning multiplication of multi-digit numbers for instance, the teachers use teachers' guide from the books published by the ministry of National Education (i.e. "MARI BERHITUNG", Moesono & Sujono, 1994) or published by a private publisher such as Erlangga, Yudistira, Surabaya (i.e. "MATEMATIKA/ BERHITUNG", 1997). The books explain the algorithm way of teaching multiplication of multi-digit numbers as follows.

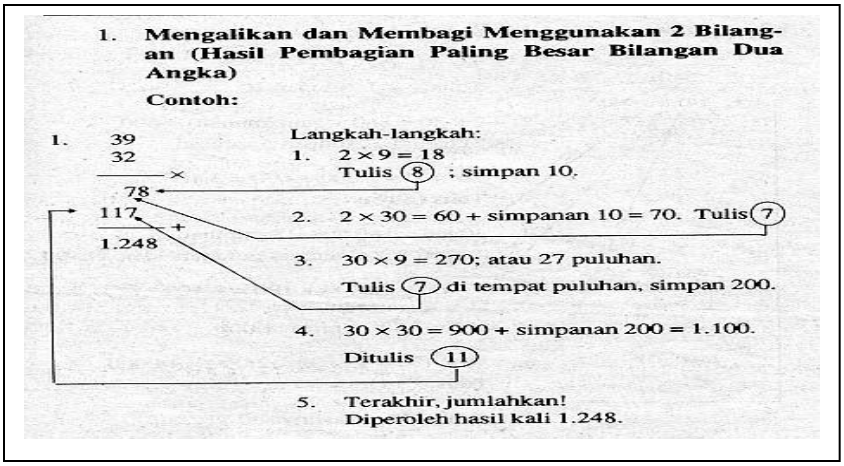

Figure 1. The algorithmic model of teaching multiplication of multi-digit numbers

The mechanistic learning begins with teachers explain each step of operations of multiplying and dividing multi-digit numbers and then the students imitate the step of operations as like the teachers did. The starting point is in the formal level of the world of symbols where the instruction becomes the presentation and drill of rules and regulations or the algorithmic mathematics education (Treffers, 1991). It is a description of teaching the algorithm mechanistically by using place value, using mental algorithm, and using standard (column) algorithm. The students learn from the teachers' explanation and imitate the strategy to solve another problem.

Using the mechanistic way of multiplying multi-digit numbers, the students memorized the operation procedures and tried to apply them in answering problem. Using their own understanding the students mistakenly applied corrupt operation procedures (Carroll & Porter, 1998) as can be seen in Figure 2 below.

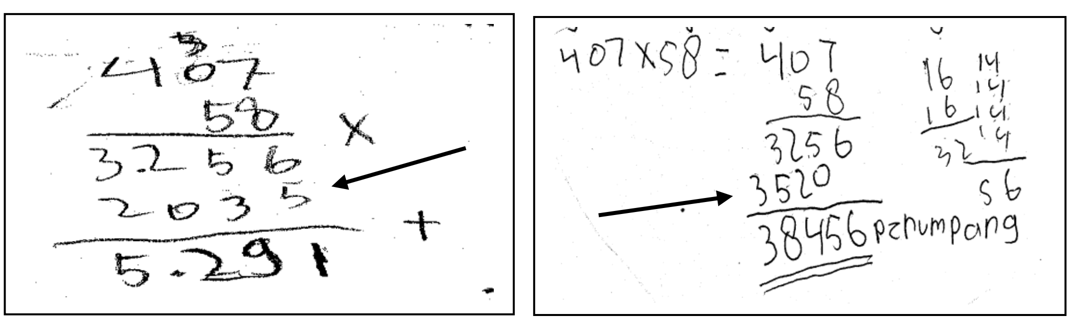

Figure 2. The corrupt multiplication procedure (*Pointed by the author)*

The mistakes showed that students had difficulty in learning multiplication mechanistically. Learning multiplication of multi-digit numbers in mechanistic approach influenced students' dependency on the teachers' explanation and they do not know other strategies because the teacher only taught the standard algorithm.

THEORETICAL ROOTS

This study is about developing, implementing, and evaluating prototypical instructional sequences for teaching multiplication of multi-digit numbers in Indonesian primary schools. The instructional sequences provide teachers opportunities to practice their teaching approach, to enlighten their knowledge, and to improve their competencies (Feiter & Van den Akker, 1995). The instructional sequences are developed based on the RME approach. Based on projects and studies in a number of countries (the Netherlands, Australia, the United Kingdom, Germany, Denmark, Japan, Malaysia, South Africa, and USA, see De Lange, 1996) it is known that the RME theory is a promising direction to improve and enhance students' understanding in mathematics. It is believed that the RME approach would be an effective approach to encounter the students' low performance problems in mathematics education in Indonesia that are caused by a number of factors, such as insufficiency of the teachers' mathematics knowledge and pedagogical approach, and the cultural aspects of the teaching and learning activity in the classroom.

The RME theory was build upon the Freudenthal argument that mathematics is a human activity, an activity of mathematizing whether subject matter from everyday-life reality or mathematical matters. Beside mathematizing the problems which are real to students, there also has to be a room for the mathematization of concepts, notations and problem-solving procedures. As a human activity, mathematics should be reinvented by the students, in which they convert a contextual problem into a mathematical problem (horizontal mathematization) and later on they structure the mathematical problem on different mathematical levels (vertical mathematization). This is called progressive mathematization (Treffers, 1987), a reinventing process of mathematical insights, knowledge, and procedures (Gravemeijer, 1997). The emphasis of this process is on allowing the students to regard the knowledge they acquire as their own, private knowledge; an understanding of which they themselves are responsible for.

The RME approach suggests facilitating the students' learning process by exploring the contextual problems in which the mathematics is embedded. Opportunity to do mathematics using their own understanding helps the students restructuring their own knowledge. Guidance from teacher supports the students reinventing informal and formal mathematics forms (concepts and procedures). These are the essential anchors of developing the students' learning attitude and understanding of mathematics.

In this study, the learning multiplication of multi-digit numbers in Grade 4 of Indonesian primary schools begins with introducing a contextual problem to encourage the students to use their understanding of repeated addition. This is the beginning of the learning trajectory towards the multiplication algorithm. In order to facilitate this RME teaching approach, the study develops instructional sequences by considering the RME theory and the Indonesian contexts. These sequences guide the teacher as well as the researcher to apply the RME approach effectively in the classroom.

RESEARCH METHODS

This study is part of the developmental research projects in which a cyclic process of front-end analysis, expert reviews, teaching experiments, and reflection to the local instructional sequences. The cyclic process leads the study to build a conjectured local instructional theory of teaching multiplication of multi-digit numbers in Indonesian primary schools. The developmental activities are illustrated as follows:

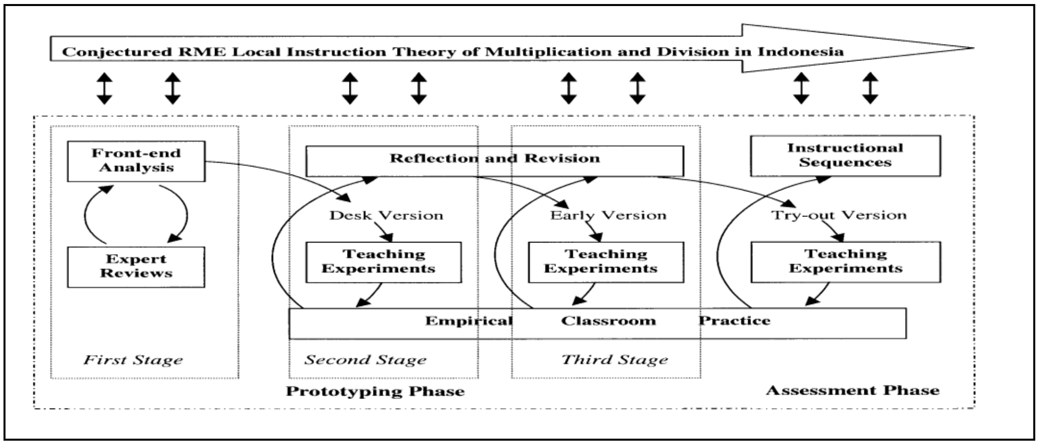

Figure 3. The developmental research activities

This study was part of the prototyping phase that focused on analyzing the validity, the practicality, and the effectiveness of the learning sequences of the RME prototype. This phase utilized various instruments (checklist, quizzes, logbooks, and tests) and different people (students, teachers, and observers) to collect the data. Validity referred to whether the learning sequences represented the Indonesian circumstances (mathematics curriculum and Indonesian conditions) as well as the RME theory. The practicality was defined as the degree to which the RME prototype was usable and easy to be applied in Indonesian primary schools. The effectiveness was defined as the degree to which the RME prototype (the learning sequence) improved the students' performances. This study analyzed and discussed the data in order to answer the following research question: *"Is the learning sequences of the RME prototype valid, practical, and effective to teach multiplication of multi-digit numbers in Indonesian primary schools?"*

To answer this question, there were two propositions to be analyzed:

1. The learning sequences of teaching multiplication of multi-digit numbers are valid and practical to be applied in Indonesian primary schools.
2. Learning multiplication of multi-digit numbers in RME approach, most students had good learning progress in daily and weekly quiz, and good achievement in solving the contextual problems in the tests (pre-test and post-test).

This study began in July 2000 until November 2000 in four primary schools in Indonesia (SD 101746 and SD 101748 klumpang, Medan, and SDN Sonosewu II and SDN Rejodadi, Yogyakarta). The subjects of the study is shown in Table 1 below. 94 students from four classes and four teachers were participated in this study which took place in the first and second tri-mester of the 2000-2001 school year. The classes were distinguished as the taught class (TC – the class that was taught by the researcher) and the observed class (OC – the class that was taught by the teacher using the RME approach and was observed by the researcher). The lessons of multiplication consisted of four units of about 80 minutes each, spread over a period of a week. These lessons were taught in two teaching experiments that were conducted in two separate time, i.e. from july 31, 2000 to August 12, 2000 in Medan and from October 30, 2000 to November 12, 2000 in Yogyakarta. Each class was given the same items of quizzes (daily and weekly quiz) and the same item of tests (pre-test and post-test). These quizzes and tests were analyzed using the progress toward the valid solution (Malone et al., 1989) to judge the effectiveness of the teaching and learning process. The teachers' logbooks and the students' portfolios were analyzed to judge the validity and practicality of the learning sequences of teaching multiplication in the RME approach.

Table 1. The subjects of the study

Schools	Type	n[a]	Teacher types	Treatment
SDN 101748 Klumpang, Medan	State	16	Novice	TC[b]
SDN Sonosewu II, Yogyakarta	State	33	Moderate	
SDN 101746 Klumpang, Medan	State	20	Experienced	OC[c]
SDN Rejodadi, Yogyakarta	State	25	Moderate	

Note: [a]Numbers of students

[b]The taught class (by the researcher) [c]The observed class (by the researcher)

RESULTS OF THE STUDY

VALIDITY AND PRACTICALITY OF THE LEARNING SEQUENCES

The learning sequences of multiplication begin with encountering contextual problems that lead students to reinvent different strategies: the repeated addition of ten numbers, the multiplication by 10, the multiplication by tens, and the standard multiplication algorithm. For these sequences the researcher developed and reevaluated various contextual problems in each day of learning multiplication. Six items were constructed for each day of teaching, three items aimed at giving opportunity for the students to understand the contents by mathematizing the problems and develop the procedures using their own learning process. The other three items served as the homework and enrichments. The items were developed and evaluated formatively considering its functions (Treffers & Goffree, 1985): model formation; concept formation; practice; applicability.

For instance, the first item in the first day of learning multiplication was "The tiles problem". The students were given a picture of a 14 x 14 square and asked to find the number of tiles to build this square. To solve this problem, it was found that some students build up several models of repeated addition, such as counting them all, addition of 5 numbers, addition of 14 numbers, and addition by ten numbers. It is a *model formation*, in which students constructed variety of models representing the "tiles" problem into mathematical forms. The

"Tiles" problem gave access and motivated students to create their own model using their own understanding. Whenever the students use the mathematical tools for finding the solution by calculating the numbers repeatedly, then it can be said that the students formatted their mathematical concepts and procedures. This calculating numbers activity supplied a firm basis for formalizing operations, notations, and rules (*concept formation*). Having many varieties of strategies (repeated addition) and discussing them with teachers and other peers would guide them toward the understanding of the use of repeated addition; the different models of solving problems; and mathematical tools being used in the calculation process. It was also found that students and teacher determined which strategy was the most understandable, and more effective and efficient to solve the problem. Then they could *practice* the strategy they were comfortable with to solve the other two problems. Solving many contextual problems that were taken from everyday life situation was intended to lead students towards their understanding of the relationship of the daily problems and the mathematics subjects; meaning that mathematics was *applicable* to solve many daily life problem situations.

These learning sequences were part of a local instructional theory for teaching multiplication of multi-digit numbers that was developed in this study. This theory was an initial Indonesian local theory that is open for adjustment and function as a guideline for others for the next developmental research study. The theory was not an ideal RME instruction. It is the intended curriculum for Indonesian circumstances that were developed based on teachers' conducting the learning activities in the classroom. Two RME experts and an Indonesian mathematics education expert evaluated these sequences to judge its validity. The experts found out that the sequences represented the Indonesian circumstances (curriculum, conditions, and culture) and the fundamental tenets of the RME theory. They, accompanied by the four teachers involved in this study, also analyzed practicality of the sequences for teaching the subject. The experts found out that the learning sequences were valid for the Indonesian primary schools. In practical teaching and learning in the classroom, the teachers found that the sequences were usable for teaching multiplication. It attracted the students to create their own understanding, to have its own strategies, to became more creative, and to be confident for themselves. However, most teachers perceived that it was not easy to apply the sequences in the classroom because of the following reasons:

1. the time available for conducting the teaching and learning was not sufficient (the experts agreed with this notion).
2. the teachers need more time to adjust the learning conditions.

THE STUDENTS' PERFORMANCES (LEARNING PROGRESS AND ACHIEVEMENT)

In the case of effectiveness, it was found that using the RME prototype in the classroom the students performed in the intended level of performances. Significant day-to-day learning progress was occurred during the learning process. Within lack of multiplication facts, 66% and 72% of students had correct answers in solving the daily quiz item and the weekly quiz item of multiplication of multi-digit numbers (see Table 2 below).

Result). The data is illustrated in the following table. Most students had good level of understanding. Using Malones' level of understanding (Malone, et al., 1989), they could proceed the problems with meaningful

Table 2. Students' correct answers in the quizzes

	Daily quiz items			Weekly quiz items	
	Multiplication			Multiplication	
Schools	1	2	3	1	2
SDN 101746	7	5	7	6	15
SDN 101748	9	6	8	8	10
SDN Rejodadi	10	13	20	11	18
SDN Sonosewu	20	20	27	18	24
Overall (%)	46	44	***62***	43	***68***
n = 94[a]	49%	47%	***66%***	46%	***72%***

Note: [a]Overall numbers of students.

work to solve the problems, however, some of them were failed to continue the calculation (see the data in Level of Approach, Substance, and

Table 3. Students' level of understanding in daily quiz for n = 94[a]

Level of understanding	Multiplication items		
	1	2	3
Non-commencement	1	1	1
Approach	8	9	3
Substance	36	35	20
Result	3	5	8
Completion	46 (49%)	44 (47%)	62 (66%)

Note: [a]Overall numbers of students.

The data in the table above indicate that most students were in the *substance level* of understanding (see the numbers in the *substance* row). It meant that the students applied sufficient detail of strategy that demonstrates they proceeded toward a rational solution, but a major error or misinterpretation obstructed the correct solution process. In the case of multiplication for instance, the major error found was whenever the students multiplied 1-digit numbers with multi-digit numbers (see Figure 4 below). Table 4 below). Table 4 bellow indicates that in the post-test the pupils' mean achievement was

25.1 and in the pre-test it was 16.3. With the mean difference of the tests was 8.8, it can be said that the pupils' score was

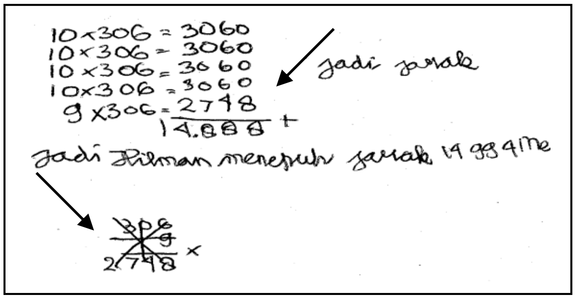

Figure 4. The students' incorrect multiplication *(pointed by author)*

In the post-test the students performed in the middle level of achievement (see

improved after engaging in the RME learning process. The post-test mean score denotes that the pupils were in the middle level of achievement.

Table 4. Pupils' average score in the pre-test and the post-test

Tests	N[a]	M	SD
Pre-test	94	16,3	6,7
Post-test	94	25,1	8,9

Note: [a]Overall numbers of pupils.

The students' achievement was improved significantly than their performance in the pre-test. As a whole the pupils' level of achievement in the pre-test and post-test is illustrated in the following table.

Table 5. Pupils' level of achievement for n = 94[a]

Score	Level	Pre-test	Post-test
0 - 13	Low	28 (30%)	11 (12%)
14 - 26	Middle	56 (59%)	36 (38%)
27 - 40	High	10 (11%)	47 (50%)

Note: [a]Overall numbers of pupils.

The table indicates that 50% of the pupils were in the high level of achievement in solving the post-test items. Meanwhile in the pre-test 70% of the pupils were in the same level. Considering this fact the study concluded that there was a considerable improvement of the pupils' achievement in solving the problems. The table also proved that the pupils' in the low ability level was decreased from 30% in the pre-test to 12% in the post-test. Most pupils progressed to the high level of achievement, which was from 10% in the pre-test to 50% in the post-test. The shift of pupils' achievement from pre-test to post-test can be seen in the following table.

Table 6. Pupils' shifting achievement from pre-test to post-test

	Post-test			
Pre-test	Low	Middle	High	∑
Low	11 (12%)	14 (15%)	3 (3%)	28 (30%)
Middle	-	22 (24%)	34 (36%)	56 (60%)
High	-	-	10 (11%)	10 (11%)
∑	11 (12%)	36 (38%)	47 (50%)	

The table indicates that 39% of the pupils (3% from the low level and 36% from the middle level) progressed to the high level of achievement. These facts showed that the pupils from the middle level got the most benefit from the RME learning process compared to the pupils from the other levels.

CONCLUSIONS AND RECOMMENDATIONS

The RME local instructional sequences developed in this study were used for teaching multiplication and division of multi-digit numbers in Indonesian primary schools. The sequences were developed based on the

This study has shown that the sequences were valid, practical (usable) in the classroom and effective for improving students' performances. Learning in the RME approach the students actively engaged in the learning activities and they performed on the expected level of understanding. This initial successful implementation experiments can be seen as inspiration for further implementation of the RME approach in Indonesian settings.

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